Solution (#1351) Let

$$I_n = \int \sec^n \theta \,\mathrm{d}\theta.$$

From Example 5.35 we have

$$I_n = \frac{\sec^{n-2}\theta \tan\theta}{n-1} + \left(\frac{n-2}{n-1}\right) I_{n-2}.$$

Hence

$$\begin{aligned}
I_9 &= \frac{\sec^7\theta\tan\theta}{8} + \frac{7}{8} I_7; \\
I_7 &= \frac{\sec^5\theta\tan\theta}{6} + \frac{5}{6} I_5; \\
I_5 &= \frac{\sec^3\theta\tan\theta}{4} + \frac{3}{4} I_3; \\
I_3 &= \frac{\sec\theta\tan\theta}{2} + \frac{1}{2} I_1.
\end{aligned}$$

Finally

$$I_1 = \ln|\sec x + \tan x| + \text{const.}.$$

So reversing these equations we find

$$\begin{aligned}
I_3 &= \frac{\sec\theta\tan\theta}{2} + \frac{1}{2}\ln|\sec x + \tan x| + \text{const.}. \\
I_5 &= \frac{\sec^3\theta\tan\theta}{4} + \frac{3\sec\theta\tan\theta}{8} + \frac{3}{8}\ln|\sec x + \tan x| + \text{const.}. \\
I_7 &= \frac{\sec^5\theta\tan\theta}{6} + \frac{5\sec^3\theta\tan\theta}{24} + \frac{15\sec\theta\tan\theta}{48} + \frac{15}{48}\ln|\sec x + \tan x| + \text{const.}. \\
I_9 &= \frac{\sec^7\theta\tan\theta}{8} + \frac{35\sec^3\theta\tan\theta}{192} + \frac{105\sec\theta\tan\theta}{384} + \frac{105}{384}\ln|\sec x + \tan x| + \text{const.}.
\end{aligned}$$