$$I_n = \int \tan^n \theta \, \mathrm{d}\theta.$$

Recalling that $\tan^2 \theta = \sec^2 \theta - 1$, we have for $n \ge 2$ that

$$I_n = \int \tan^{n-2} \theta (\sec^2 \theta - 1) d\theta$$
$$= \int \tan^{n-2} \theta \sec^2 \theta d\theta - I_{n-2}$$
$$= \frac{\tan^{n-1} \theta}{n-1} - I_{n-2}.$$