Solution (#1364) If we employ IBP then

$$\begin{aligned}
I_n &= \int_0^1 1 \times \frac{1}{(4-x^2)^n}\,\mathrm{d}x \\
&= \left[x \times \frac{1}{(4-x^2)^n}\right]_0^1 - \int_0^1 x \times \frac{(-n)(-2x)}{(4-x^2)^{n+1}}\,\mathrm{d}x \\
&= \frac{1}{3^n} - 2n\int_0^1 \frac{x^2}{(4-x^2)^{n+1}}\,\mathrm{d}x \\
&= \frac{1}{3^n} - 2n\int_0^1 \frac{x^2-4+4}{(4-x^2)^{n+1}}\,\mathrm{d}x \\
&= \frac{1}{3^n} - 2n\left(4I_{n+1} - I_n\right).
\end{aligned}$$

Rearranging this becomes

$$I_{n+1} = \frac{1}{8n3^n} + \left(\frac{2n-1}{8n}\right) I_n \qquad \text{for } n \geqslant 1.$$