Solution (#1396) Use $1-\cos x = 2\sin^2(x/2)$ and find in due course

$$\begin{aligned}\int_0^{\pi} \frac{x^2\,\mathrm{d}x}{1-\cos x} &= 4\int_0^{\pi/2} u^2 \csc^2 u\,\mathrm{d}u \\ &= -8\int_0^{\pi/2} \ln\sin u\,\mathrm{d}u \\ &= 4\pi\ln 2. \qquad [\#1395]\end{aligned}$$

Using again $1-\cos x = 2\sin^2(x/2)$ we have

$$\begin{aligned}\int_0^{\pi} \frac{x-\sin x}{1-\cos x}\,\mathrm{d}x &= 2\int_0^{\pi/2} u\csc^2 u - \cot u\,\mathrm{d}u \\ &= 2\left[u(-\cot u)\right]_0^{\pi/2} \\ &= 2\lim_{u\to 0} u\cot u = 2\end{aligned}$$

noting from #1286 that

$$\lim_{u\to 0} u\cot u = \lim_{u\to 0} u\frac{\cos u}{\sin u} = \left(\lim_{u\to 0}\frac{u}{\sin u}\right)\left(\lim_{u\to 0}\cos u\right) = 1\times 1 = 1.$$