Solution (#1406) Recall that

$$B\left(\frac{3}{2},\frac{3}{2}\right)=\int_0^1\sqrt{x\,(1-x)}\,\mathrm{d}x.$$

Note that $x=\frac{1}{2}(1-\sin\theta)$ is decreasing on $-\pi/2\leqslant\theta\leqslant\pi/2$. We have

$$\mathrm{d}x=-\frac{1}{2}\cos\theta\,\mathrm{d}\theta,$$

so that the above integral becomes

$$\begin{aligned}
&\int_{\pi/2}^{-\pi/2}\sqrt{\left(\frac{1-\sin\theta}{2}\right)\left(\frac{1+\sin\theta}{2}\right)}\left(-\frac{1}{2}\cos\theta\,\mathrm{d}\theta\right)\\
=\;&\frac{1}{4}\int_{-\pi/2}^{\pi/2}\sqrt{\cos^2\theta}\,\cos\theta\,\mathrm{d}\theta\\
=\;&\frac{1}{4}\int_{-\pi/2}^{\pi/2}\cos^2\theta\;\mathrm{d}\theta\\
=\;&\frac{1}{8}\int_{-\pi/2}^{\pi/2}(1+\cos2\theta)\;\mathrm{d}\theta\\
=\;&\frac{1}{8}\left[\theta+\frac{1}{2}\sin2\theta\right]_{-\pi/2}^{\pi/2}\\
=\;&\frac{\pi}{8}.
\end{aligned}$$