Solution (#1409) By definition

$$B(a+1,-a)=\int_0^1 x^a(1-x)^{-a-1}\,\mathrm{d}x.$$

To relate $B(a+1,-a)$ to the second integral set $u=x/(1-x)$.
We can relate the second integral to the third by IBP.
Finally to relate the second integral to the fourth, we set $x=\tan^2 t$.
Looking at the last integral we see that when $a=-1/2$ we have $B(1/2,1/2)=\pi$.