Solution (#1412) We have from #1411 that

$$B(a,b) = \int_{u=0}^{u=\infty} \frac{u^{a-1}}{(1+u)^{a+b}}\ \mathrm{d}u.$$

(i) If we set $u = x^n$ so that $\mathrm{d}x = \frac{1}{n}u^{1/n-1}\mathrm{d}u$ then

$$\int_0^\infty \frac{\mathrm{d}x}{(x^n+1)^m} = \frac{1}{n}\int_0^\infty \frac{u^{1/n-1}\,\mathrm{d}u}{(1+u)^m} = \frac{1}{n}B\left(\frac{1}{n}, m - \frac{1}{n}\right).$$

(ii) If we set $u = x^n$ again then

$$\begin{aligned}\int_0^\infty \frac{x^m\,\mathrm{d}x}{\sqrt{1+x^n}} &= \frac{1}{n}\int_0^\infty \frac{u^{m/n}\,u^{1/n-1}\,\mathrm{d}u}{\sqrt{1+u}} \\ &= \frac{1}{n}\int_0^\infty \frac{u^{(m+1)/n-1}\,\mathrm{d}u}{(1+u)^{1/2}} \\ &= \frac{1}{n}B\left(\frac{m+1}{n}, \frac{1}{2} - \left(\frac{m+1}{n}\right)\right).\end{aligned}$$