Solution (#1421) Note that for any natural number k then

$$\int_{k\pi}^{(k+1)\pi} \left| \frac{\sin x}{x} \right| \mathrm{d}x \geqslant \int_{k\pi}^{(k+1)\pi} \left| \frac{\sin x}{(k+1)\pi} \right| \mathrm{d}x = \frac{2}{(k+1)\pi}.$$

Hence dividing the integral up into integrals on $[0, \pi)$, $[\pi, 2\pi)$, $[2\pi, 3\pi)$,... we find

$$\int_0^{n\pi} \left| \frac{\sin x}{x} \right| \mathrm{d}x \geqslant \frac{2}{\pi} + \frac{2}{2\pi} + \frac{2}{3\pi} + \cdots + \frac{2}{n\pi} = \frac{2}{\pi} H_n,$$

where H_n denotes the nth harmonic number. By #1325 we know the harmonic series to increase without bound and hence the integral

$$\int_0^{\infty} \left| \frac{\sin x}{x} \right| \mathrm{d}x$$

does not converge.