Solution (#1445) Set $x = \tan^{-1} u^2$ to find

$$\int \sqrt{\tan x}\ \mathrm{d}x = \int \frac{2u^2\ \mathrm{d}u}{1+u^4} = \frac{1}{\sqrt{2}} \int \left(\frac{u}{u^2+1-\sqrt{2}u} - \frac{u}{u^2+1+\sqrt{2}u} \right) \mathrm{d}u$$

The answer is

$$\int \sqrt{\tan x}\ \mathrm{d}x = \frac{1}{2\sqrt{2}} \left\{ \ln \left| \frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1} \right| + 2\tan^{-1} \left(\frac{\sqrt{2\tan x}}{1-\tan x} \right) \right\} + \mathrm{const.}$$