Solution (#1453) Say that $f(x)$ is a convex function, so that $f''(x) \geqslant 0$ for all x. From #1334 we have on any interval $[x_i, x_{i+1}]$ that

$$f\left(tx_i + (1-t)x_{i+1}\right) \leqslant ty_i + (1-t)y_{i+1}.$$

Integrating the RHS we get

$$\int_0^1 ty_i + (1-t)y_{i+1}\,\mathrm{d}t = \frac{y_i + y_{i+1}}{2},$$

and integrating the LHS we get

$$\begin{aligned}
& \int_0^1 f\left(tx_i + (1-t)x_{i+1}\right)\,\mathrm{d}t \\
= & \int_0^1 f\left(x_i + (1-t)(x_{i+1} - x_i)\right)\,\mathrm{d}t \\
= & \int_0^1 f\left(x_i + (1-t)h\right)\,\mathrm{d}t \\
= & \int_{x_{i+1}}^{x_i} f(u)\left(\frac{\mathrm{d}u}{-h}\right) \qquad [u = x_i + (1-t)h] \\
= & \frac{1}{h}\int_{x_i}^{x_{i+1}} f(u)\,\mathrm{d}u.
\end{aligned}$$

So we have

$$\int_{x_i}^{x_{i+1}} f(u)\,\mathrm{d}u \leqslant \left(\frac{y_i + y_{i+1}}{2}\right)h$$

and see that the trapezium rule provides an overestimate of the interval on each subinterval, and hence overall gives an overestimate.