**Solution** (#1459) (i) Consider the function $y = x^4$ between $0 \leqslant x \leqslant 1$. If we use two intervals then we get the two estimates

$$\text{trapezium:} \qquad \frac{1/2}{2}\left(0^4 + 2\frac{1}{2^4} + 1^4\right) = \frac{9}{32};$$

$$\text{simpson's:} \qquad \frac{1/2}{3}\left(0^4 + 4\frac{1}{2^4} + 1^4\right) = \frac{5}{24}.$$

As the correct answer is $1/5$ then the trapezium rule and Simpson's rule both give overestimates.

(ii) If instead we choose $y = -x^4$ between $0 \leqslant x \leqslant 1$ with two intervals then the two rules both give underestimates.

(iii) Now set $f(x) = \sin x$ between $0 \leqslant x \leqslant \pi/2$ with two intervals. We get the estimates

$$\text{trapezium:} \qquad \frac{\pi/4}{2}\left(0 + 2\frac{1}{\sqrt{2}} + 1\right) = \frac{\pi}{8}(1+\sqrt{2}) = 0.948\ldots;$$

$$\text{Simpson's:} \qquad \frac{\pi/4}{3}\left(0 + 4\frac{1}{\sqrt{2}} + 1\right) = \frac{\pi}{12}(1+2\sqrt{2}) = 1.002\ldots.$$

The correct answer is 1 and so the trapezium rule gives an underestimate and Simpson's rule an overestimate.

(iv) Setting $f(x) = -\sin x$ between $0 \leqslant x \leqslant \pi/2$ with two intervals we get an overestimate using the trapezium rule and an underestimate from Simpson's.