

Solution (#1464) Stirling's approximation is always an underestimate of the factorial. We see with a computer that

$$\begin{aligned} \text{(i)} \quad & \frac{\sqrt{2\pi n}(n/e)^n}{n!} \geq 0.9 \quad \text{for } n \geq 1; \\ \text{(ii)} \quad & \frac{\sqrt{2\pi n}(n/e)^n}{n!} \geq 0.99 \quad \text{for } n \geq 9; \\ \text{(iii)} \quad & \frac{\sqrt{2\pi n}(n/e)^n}{n!} \geq 0.999 \quad \text{for } n \geq 84; \\ \text{(iv)} \quad & \frac{\sqrt{2\pi n}(n/e)^n}{n!} \geq 0.9999 \quad \text{for } n \geq 834; \end{aligned}$$