Solution (#1470) Let a, b be real constants with $b > 0$. Then setting $x + a = b \tan t$ we find

$$\int_0^\infty \frac{\mathrm{d}x}{(x+a)^2 + b^2} = \int_{\tan^{-1}(a/b)}^{\pi/2} \frac{b \sec^2 t \, \mathrm{d}t}{b^2 \tan^2 t + b^2} = \frac{1}{b} \int_{\tan^{-1}(a/b)}^{\pi/2} \mathrm{d}t = \frac{1}{b} \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{a}{b} \right) \right) = \frac{1}{b} \cot^{-1} \left(\frac{a}{b} \right).$$