

Solution (#1475) If we substitute $x = e^{-t}$ then

$$\begin{aligned}\int_0^1 \frac{x-1}{\ln x} dx &= \int_{\infty}^0 \frac{e^{-t}-1}{-t} (-e^{-t} dt) \\ &= \int_0^{\infty} \frac{e^{-t}-e^{-2t}}{t} dt \\ &= \ln\left(\frac{2}{1}\right) \\ &= \ln 2,\end{aligned}$$

as this last integral was evaluated in #1329.