Solution (#1480) We begin with IBP to see

$$\int_0^{\pi/2} x \cot x \, \mathrm{d}x = [x \ln \sin x]_0^{\pi/2} - \int_0^{\pi/2} \ln \sin x \, \mathrm{d}x.$$

We have already seen in #1286 and #1318 that

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \qquad \text{and} \qquad \lim_{x \to 0} x \ln x = 0$$

so that

$$\lim_{x \to 0} x \ln \sin x = \lim_{x \to 0} \left(x \ln \frac{\sin x}{x} + x \ln x \right) = 0 + 0 = 0.$$

So

$$\int_0^{\pi/2} x \cot x \, \mathrm{d}x = -\int_0^{\pi/2} \ln \sin x \, \mathrm{d}x = \frac{\pi}{2} \ln 2$$

as this integral was determined in #1395.