

Solution (#1489) Note that

$$x^2 \left(\frac{1}{1-e^x} - \frac{1}{2} \right) = \frac{x^2}{2} \left(\frac{1+e^x}{1-e^x} \right) = \frac{x^2}{2} \left(\frac{e^{-x/2} + e^{x/2}}{e^{-x/2} - e^{x/2}} \right) = -\frac{x^2 \cosh(x/2)}{2 \sinh(x/2)}$$

is odd and hence the integral, being from $-\pi$ to π , equals zero.