Solution (#1492) Let $a, b > 0$. If we set $u = \sin^2 x$ then we have $x = \sin^{-1} \sqrt{u}$ and so

$$\mathrm{d}x = \frac{1}{\sqrt{1-u}} \frac{\mathrm{d}u}{2\sqrt{u}}$$

and so

$$
\begin{aligned}
\int_0^{\pi/2} \sin^a x \cos^b x \, \mathrm{d}x &= \int_0^1 u^{a/2}(1-u)^{b/2} \left(\frac{1}{\sqrt{1-u}} \frac{\mathrm{d}u}{2\sqrt{u}} \right) \\
&= \frac{1}{2} \int_0^1 u^{(a-1)/2}(1-u)^{(b-1)/2} \, \mathrm{d}u \\
&= \frac{1}{2} B\left(\frac{a-1}{2} + 1, \frac{b-1}{2} + 1 \right) \\
&= \frac{1}{2} B\left(\frac{a+1}{2}, \frac{b+1}{2} \right).
\end{aligned}
$$