Solution (#1503) Let $a > 1$. Set

$$I = \int_{-\infty}^{\infty} \frac{\mathrm{d}x}{(x^2+1)\sqrt{x^2+a^2}}.$$

If we set $x = a \tan u$ then we find

$$\begin{aligned} I &= \int_{-\pi/2}^{\pi/2} \frac{\sec u \, \mathrm{d}u}{a^2 \tan^2 u + 1} \\ &= \int_{-\pi/2}^{\pi/2} \frac{\cos u \, \mathrm{d}u}{a^2 \sin^2 u + \cos^2 u} \\ &= \int_{-\pi/2}^{\pi/2} \frac{\cos u \, \mathrm{d}u}{1 + (a^2-1)\sin^2 u} \\ &= \frac{1}{a^2-1} \int_{-1}^{1} \frac{\mathrm{d}v}{\frac{1}{a^2-1} + v^2} \qquad [v = \sin u] \\ &= \frac{1}{a^2-1} \sqrt{a^2-1} \left[\tan^{-1}\left(\sqrt{a^2-1}\,v\right) \right]_{-1}^{1} \\ &= \frac{2}{\sqrt{a^2-1}} \tan^{-1} \sqrt{a^2-1}. \end{aligned}$$