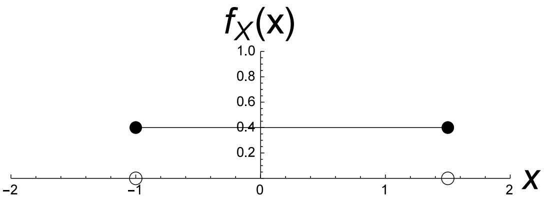
Solution (#1513) Let $a < b$. If X has the uniform distribution on $[a, b]$ then the cdf is

$$F_X(x) = P(X \leqslant x) = \frac{1}{b-a} \int_{-\infty}^{x} \mathbf{1}_{[a,b]}(t)\,\mathrm{d}t = \begin{cases} 0 & x < a; \\ \frac{x-a}{b-a} & a \leqslant x \leqslant b; \\ 1 & b < x. \end{cases}$$

Solution



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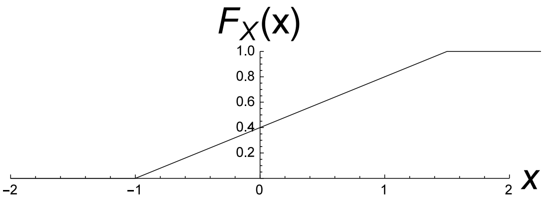

cdf

The expectation $E(X)$ equals

$$\frac{1}{b-a} \int_{-\infty}^{\infty} t\,\mathbf{1}_{[a,b]}(t)\,\mathrm{d}t = \frac{1}{b-a} \int_{a}^{b} t\,\mathrm{d}t = \frac{1}{b-a} \left[\frac{t^2}{2}\right]_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{b+a}{2}.$$

The variance $\mathrm{Var}(X)$ equals

$$\begin{aligned} & \left(\frac{1}{b-a} \int_a^b t^2\,\mathrm{d}t\right) - \left(\frac{a+b}{2}\right)^2 \\ = \; & \frac{(b^3 - a^3)}{3(b-a)} - \left(\frac{a+b}{2}\right)^2 \\ = \; & \left(\frac{b^2 + ab + a^2}{3}\right) - \left(\frac{a^2 + 2ab + b^2}{4}\right) \\ = \; & \frac{b^2 - 2ab + a^2}{12} \\ = \; & \frac{1}{12}\,(b-a)^2. \end{aligned}$$

and variance.