Solution (#1520) Let $f_X(x)$ be the pdf of a continuous random variable. The median of

(i) $(b-a)^{-1}\mathbf{1}_{(a,b)}(x)$ is

$$m = \frac{a+b}{2}.$$

As

$$\int_a^{(a+b)/2} \frac{\mathrm{d}x}{b-a} = \frac{1}{2}.$$

(ii) To find the median of the exponential we need

$$\int_0^m \lambda e^{-\lambda x}\,\mathrm{d}x = 1 - e^{-\lambda m} = \frac{1}{2}$$

and so

$$m = \frac{\ln 2}{\lambda}.$$

(iii) The median of the normal distribution with parameters μ and σ^2 is again μ (the same as the mean) as

$$\begin{aligned} & \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\mu} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \mathrm{d}x \\ = & \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{0} \exp\left(-\frac{u^2}{2\sigma^2}\right) \mathrm{d}u \qquad [u = x-\mu] \\ = & \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{0} \exp\left(-v^2\right) \sqrt{2}\sigma\,\mathrm{d}v \qquad [u = \sqrt{2}\sigma v] \\ = & \frac{\sqrt{2}\sigma}{\sqrt{2\pi\sigma^2}} \frac{\sqrt{\pi}}{2} = \frac{1}{2}. \end{aligned}$$

(iv) The Cauchy distribution has median $m = 0$ as

$$\int_{-\infty}^{0} \frac{\mathrm{d}x}{\pi(1+x^2)} = \frac{1}{2}.$$