Solution (#1544) We can parametrize a quarter of the ellipse by setting

$$(x(t), y(t)) = (a \sin t, b \cos t) \qquad 0 \leqslant t \leqslant \pi/2.$$

Now

$$\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = (a \cos t)^2 + (-b \sin t)^2 = a^2 \cos^2 t + b^2 \sin^2 t.$$

Hence the length of the ellipse equals

$$4 \int_0^{\pi/2} \sqrt{a^2 \cos^2 t + b^2 \sin^2 t}\, \mathrm{d}t.$$

If $a \geqslant b$ then we can rewrite this as

$$\begin{aligned} & 4 \int_0^{\pi/2} \sqrt{a^2 + (b^2 - a^2) \sin^2 t}\, \mathrm{d}t \\ = \quad & 4a \int_0^{\pi/2} \sqrt{1 - \left(1 - \frac{b^2}{a^2}\right) \sin^2 t}\, \mathrm{d}t \\ = \quad & 4aE\left(\sqrt{1 - \frac{b^2}{a^2}}\right). \end{aligned}$$