Solution (#1546) (i) Let $a > b > 0$. Using $\cos x = 1 - 2\sin^2 \frac{x}{2}$ we see

$$\begin{aligned}
&\int_0^\pi \sqrt{a + b\cos x}\,\mathrm{d}x \\
= &\int_0^\pi \sqrt{a + b - 2b\sin^2 \frac{x}{2}}\,\mathrm{d}x \\
= &\int_0^{\pi/2} \sqrt{a + b - 2b\sin^2 u}\,2\,\mathrm{d}u \qquad [u = x/2] \\
= &\ 2\sqrt{a+b}\int_0^{\pi/2} \sqrt{1 - \frac{2b}{a+b}\sin^2 u}\,\mathrm{d}u \\
= &\ 2\sqrt{a+b}E\left(\sqrt{\frac{2b}{a+b}}\right).
\end{aligned}$$

(ii) Again using $\cos x = 1 - 2\sin^2 \frac{x}{2}$ we see

$$\begin{aligned}
&\int_0^\pi \frac{\mathrm{d}x}{\sqrt{a + b\cos x}} \\
= &\int_0^\pi \frac{\mathrm{d}x}{\sqrt{a + b - 2b\sin^2 \frac{x}{2}}} \\
= &\int_0^{\pi/2} \frac{2\,\mathrm{d}u}{\sqrt{a + b - 2b\sin^2 u}} \qquad [u = x/2] \\
= &\ \frac{2}{\sqrt{a+b}}\int_0^{\pi/2} \frac{\mathrm{d}u}{\sqrt{1 - \frac{2b}{a+b}\sin^2 u}} \\
= &\ \frac{2}{\sqrt{a+b}}K\left(\sqrt{\frac{2b}{a+b}}\right).
\end{aligned}$$