Solution (#1571) Recall from #1570 that

$$\Gamma(a) = \int_0^\infty t^{a-1}e^{-t}\,\mathrm{d}t, \qquad \Gamma'(a) = \int_0^\infty t^{a-1}e^{-t}\ln t\,\mathrm{d}t.$$

We similarly see that the second derivative equals

$$\Gamma''(a) = \int_0^\infty t^{a-1}e^{-t}(\ln t)^2\,\mathrm{d}t > 0$$

and is always positive because the integrand is positive. Consequently $\Gamma'(a)$ is an increasing function for $a > 0$ and can have at most one root in that range.

From #1326 we know that $0 < \gamma < 1$. As

$$\Gamma'(1) = -\gamma < 0$$

and

$$\Gamma'(2) = \psi(2)\Gamma(2) = (1-\gamma)\times 1 > 0$$

then (by a continuity argument) the root of $\Gamma'(a)$ lies in the interval $1 < a < 2$.