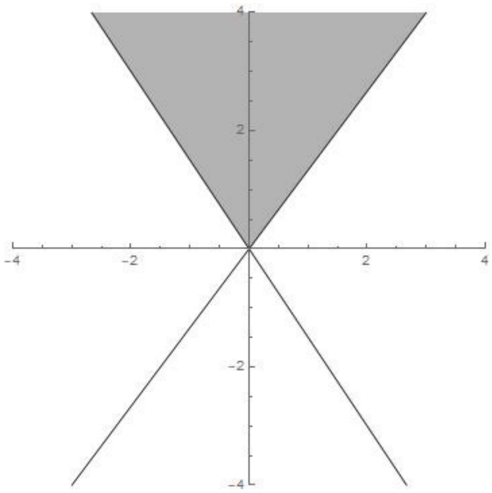
Solution (#490) Let $\mathbf{v} = (3, 4)$ and $\mathbf{w} = (2, -3)$ in $\mathbb{R}^2$.
(i) Let $\mathbf{x} = (x, y) = \alpha\mathbf{v} + \beta\mathbf{w}$. Then

$$\alpha = \frac{3x + 2y}{17} \quad \text{and} \quad \beta = \frac{4x - 3y}{17}.$$

in $\mathbb{R}^2$.
(ii) The region $\{\alpha\mathbf{v} + \beta\mathbf{w} : \alpha > 0 > \beta\}$ is the region $\left\{(x, y) \in \mathbb{R}^2 : 3x + 2y > 0 \ \text{ and } \ 4x < 3y\right\}$ as sketched below



(iii) If we take $\mathbf{s} = (1, 0)$, and $\mathbf{t} = (2, 0)$ then $\{\alpha\mathbf{s} + \beta\mathbf{t} \colon \alpha, \beta \in \mathbb{R}\}$ is the x-axis. It will be more generally be the case that $\{\alpha\mathbf{s} + \beta\mathbf{t} \colon \alpha, \beta \in \mathbb{R}\} \neq \mathbb{R}^2$ if one vector is a scalar multiple of the other.