Solution (#491) Let $\mathbf{v} = (1,0,2)$, $\mathbf{w} = (0,3,1)$ in $\mathbb{R}^3$. Suppose that $(x,y,z) = \alpha\mathbf{v} + \beta\mathbf{w}$ for some real numbers α, β. Then this gives us three scalar equations

$$x = \alpha, \qquad y = 3\beta, \qquad z = 2\alpha + \beta.$$

Hence, eliminating α and β, we have

$$z = 2x + y/3 \quad \text{which rearranges to} \quad 6x + y - 3z = 0.$$

Conversely if $6x + y - 3z = 0$, then

$$(x,y,z) = (x, 3z - 6x, z) = x\,(1,0,2) + (z - 2x)\,(0,3,1)$$

which is the required result with $\alpha = x$ and $\beta = z - 2x$.