

Solution (#503) We have

$$\mathbf{v} = (|\mathbf{v}| \cos(\alpha + \beta), |\mathbf{v}| \sin(\alpha + \beta)) \quad \text{and} \quad \mathbf{w} = (|\mathbf{w}| \cos \alpha, |\mathbf{w}| \sin \alpha).$$

Then

$$\begin{aligned} \mathbf{v} \cdot \mathbf{w} &= |\mathbf{v}| |\mathbf{w}| (\cos(\alpha + \beta) \cos \alpha + \sin(\alpha + \beta) \sin \alpha) \\ &= |\mathbf{v}| |\mathbf{w}| \cos(\alpha + \beta - \alpha) \\ &= |\mathbf{v}| |\mathbf{w}| \cos(\beta). \end{aligned}$$

Hence

$$\cos^{-1} \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|} \right) = \beta$$

as required.