Solution (#507) If $\mathbf{u} = (x, y, z)$ is a unit vector in $\mathbb{R}^3$ then $-1 \leqslant z \leqslant 1$ and so there is a unique θ in the range $0 \leqslant \theta \leqslant \pi$ such that $z = \cos\theta$. For $\sin\theta \neq 0$ we then have

$$\left(\frac{x}{\sin\theta}\right)^2 + \left(\frac{y}{\sin\theta}\right)^2 = 1$$

and so there is a unique ϕ in the range $0 \leqslant \phi < 2\pi$ such that

$$(x/\sin\theta, y/\sin\theta) = (\cos\phi, \sin\phi).$$

The case of $\sin\theta = 0$ needs treating separately.