Solution (#526) If $\lambda = \mathbf{r} \cdot \mathbf{a}$ and $\mu = \mathbf{r} \cdot \mathbf{b}$ hold true for all points of the plane, then

$$\lambda = \lambda \mathbf{a} \cdot \mathbf{a} + \mu \mathbf{b} \cdot \mathbf{a} \qquad \text{for all } \lambda, \mu.$$

Now take $\lambda = 1, \mu = 0$ and take $\lambda = 0, \mu = 1$. Similarly dot $\mathbf{r} = \lambda \mathbf{a} + \mu \mathbf{b}$ with $\mathbf{b}$.

Conversely if $|\mathbf{a}| = 1$, $|\mathbf{b}| = 1$, $\mathbf{a} \cdot \mathbf{b} = 0$, then separately dot $\mathbf{r} = \lambda \mathbf{a} + \mu \mathbf{b}$ with $\mathbf{a}$ and with $\mathbf{b}$.