Solution (#556) We have

$$A = \left(i+j^2\right) = \begin{pmatrix} 2 & 5 & 10 \\ 3 & 6 & 11 \\ 4 & 7 & 12 \end{pmatrix}; \qquad B = (i-j) = \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}.$$

Then

$$\begin{aligned} AB &= \begin{pmatrix} 2 & 5 & 10 \\ 3 & 6 & 11 \\ 4 & 7 & 12 \end{pmatrix}\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0+5+20 & -2+0+10 & -4-5+0 \\ 0+6+22 & -3+0+11 & -6-6+0 \\ 0+7+24 & -4+0+12 & -8-7+0 \end{pmatrix} = \begin{pmatrix} 25 & 8 & -9 \\ 28 & 8 & -12 \\ 31 & 8 & -15 \end{pmatrix}; \\ BA &= \begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 5 & 10 \\ 3 & 6 & 11 \\ 4 & 7 & 12 \end{pmatrix} = \begin{pmatrix} 0-3-8 & 0-6-14 & 0-11-24 \\ 2+0-4 & 5+0-7 & 10+0-12 \\ 4+3+0 & 10+6+0 & 20+11+0 \end{pmatrix} = \begin{pmatrix} -11 & -20 & -35 \\ -2 & -2 & -2 \\ 7 & 16 & 31 \end{pmatrix}. \end{aligned}$$

Now

$$\begin{aligned} [AB]_{ij} &= \sum_{k=1}^{3} [A]_{ik}[B]_{kj} \\ &= \sum_{k=1}^{3} (i+k^2)(k-j) \\ &= (i+1)(1-j) + (i+4)(2-j) + (i+9)(3-j) \\ &= \{1+i-j-ij\} + \{8+2i-4j-ij\} + \{27+3i-9j-ij\} \\ &= -3ij + 6i - 14j + 36, \end{aligned}$$

and similarly

$$\begin{aligned} [BA]_{ij} &= \sum_{k=1}^{3} [B]_{ik}[A]_{kj} \\ &= \sum_{k=1}^{3} (i-k)(k+j^2) \\ &= (i-1)(1+j^2) + (i-2)(2+j^2) + (i-3)(3+j^2) \\ &= \left\{ij^2 + i - j^2 - 1\right\} + \left\{ij^2 + 2i - 2j^2 - 4\right\} + \left\{ij^2 + 3i - 3j^2 - 9\right\} \\ &= 3ij^2 + 6i - 6j^2 - 14. \end{aligned}$$