Solution (#567) We note

$$\sum_{j=1}^{n} \delta_{ij} = \delta_{ii} = 1;$$

and using this in the second sum we get

$$\sum_{i=1}^{n} \left(\sum_{j=1}^{n} \delta_{ij} \right) = \sum_{i=1}^{n} 1 = n$$

or we could simply note $\delta_{ij} = 1$ on n occasions when $i = j$. For the third sum

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \delta_{kj} \delta_{ki} = \sum_{i=1}^{n} \delta_{ki} \left(\sum_{j=1}^{n} \delta_{kj} \right) = \sum_{i=1}^{n} (\delta_{ki} \times 1) = 1$$

or again we can note $\delta_{kj}\delta_{ki} = 1$ in the single instance of $(i, j) = (k, k)$. For the final sum

$$\sum_{k=1}^{n} \delta_{kj} \delta_{ki} = \sum_{k=1}^{n} [I_n]_{kj} [I_n]_{ki} = \sum_{k=1}^{n} \left[I_n^T \right]_{ik} [I_n]_{kj} = \left[I_n^T \, I_n \right]_{ij} = [I_n]_{ij} = \delta_{ij}$$

or one could simply argue

$$\sum_{k=1}^{n} \delta_{kj} \delta_{ki} = \delta_{jj} \delta_{ji} = \delta_{ij}.$$