

**Solution** (#615) Let  $A$  be a square matrix. Suppose that  $A$  is invertible. Then by the product rule for transposes

$$(A^{-1})^T A^T = (AA^{-1})^T = I^T = I;$$

$$A^T (A^{-1})^T = (A^{-1}A)^T = I^T = I,$$

and so  $A^T$  is invertible with inverse  $(A^{-1})^T$ .

Conversely say that  $A^T$  is invertible. By the above  $(A^T)^T = A$  is invertible.