Solution (#628) Suppose that $A = P^{-1}BP$.
(i) Then
$$A^2 = (P^{-1}BP)(P^{-1}BP) = P^{-1}B^2P.$$
Hence A^2 is similar to B^2 (with the same choice of P).
(ii) We also have
$$\text{trace}(A) = \text{trace}(P^{-1}(BP)) = \text{trace}((BP)P^{-1}) = \text{trace}(B).$$
(iii) Finally
$$A^T = (P^{-1}BP)^T = P^TB^T(P^{-1})^T = ((P^{-1})^T)^{-1}B^T(P^{-1})^T.$$
So A^T and B^T are similar (with generally a different choice of P unless $(P^{-1})^T = P$, i.e. unless P is orthogonal).