Solution (#634) Let

$$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad B=\begin{pmatrix} e & f \\ g & h \end{pmatrix},$$

so that

$$AB=\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}=\begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}.$$

Then

$$\begin{aligned} \det(AB) &= (ae+bg)(cf+dh)-(af+bh)(ce+dg) \\ &= (acef+bdgh+bcfg+adeh)-(acef+bceh+adfg+bdgh) \\ &= bcfg+adeh-bceh-adfg \\ &= (ad-bc)(eh-fg) \\ &= \det A \det B. \end{aligned}$$

The A in Example 3.52 has determinant

$$\det\begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix}=-\cos^2\alpha-\sin^2\alpha=-1.$$

If A had a square root M then we would have

$$\det A=\det(M^2)=(\det M)^2\geqslant 0.$$

Hence there is no (real) matrix M such that $M^2=A$.