Solution (#650) The system reduces as

$$\begin{pmatrix} 1 & 2 & \alpha^2 \\ 1 & \alpha & 1 \\ 1 & \alpha & \alpha^2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & \alpha^2 \\ 0 & \alpha-2 & 0 \\ 0 & 0 & 1-\alpha^2 \end{pmatrix}.$$

If and only if $\alpha = 2, 1, -1$, then the system has infinitely many solutions.

$$\alpha = 2\colon \quad \begin{pmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix} \quad \text{general solution } (-2k, k, 0)\,;$$

$$\alpha = 1\colon \quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{general solution } (-k, 0, k)\,;$$

$$\alpha = -1\colon \quad \begin{pmatrix} 1 & 2 & 1 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{general solution } (-k, 0, k)\,.$$