

Solution (#652) On reducing the system we find

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ a & 1 & 2 & 2 \\ 1 & a & 1 & 4 \end{array} \right) \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & a \\ 0 & 1-a & 2-a & 2-a^2 \\ 0 & 0 & 2-a & 6-a-a^2 \end{array} \right).$$

- (i) If $a \neq 1, 2$ the system has a unique solution.
- (ii) If $a = 1$ the system is inconsistent.
- (iii) If $a = 2$ the general solution is $(-t, 2, t)$ where $t \in \mathbb{R}$.