**Solution** (#689) Column-reduction and row-reduction of the first matrix gives

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & 0 & 8/7 \\ 0 & 1 & 0 & 3/7 \\ 0 & 0 & 1 & 3/7 \end{pmatrix}.$$

Note that the row rank and column rank both equal three.

Column-reduction and row-reduction of the second matrix gives

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3/7 & -1/7 & 2/7 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Note that the row rank and column rank both equal three.

Column-reduction and row-reduction of the third matrix gives

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & -3/2 & 1/2 \\ 0 & 1 & 9/4 & 1/4 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Note that the row rank and column rank both equal two.