**Solution** (#696) Suppose that $\mathbf{v}_i$ can be written as a linear combination of $\mathbf{v}_1, \ldots, \mathbf{v}_{i-1}, \mathbf{v}_{i+1} \ldots, \mathbf{v}_m$ so that

$$\mathbf{v}_i = \lambda_1 \mathbf{v}_1 + \cdots + \lambda_{i-1}\,\mathbf{v}_{i-1} + \lambda_{i+1}\mathbf{v}_{i+1} + \cdots + \lambda_m \mathbf{v}_m.$$

Then

$$\lambda_1 \mathbf{v}_1 + \cdots + \lambda_{i-1}\,\mathbf{v}_{i-1} - \mathbf{v}_i + \lambda_{i+1}\mathbf{v}_{i+1} + \cdots + \lambda_m \mathbf{v}_m = \mathbf{0}$$

without all the scalars being zero and hence $\mathbf{v}_1,\ \mathbf{v}_2, \ldots, \mathbf{v}_m$ are dependent.

Conversely suppose that $\mathbf{v}_1,\ \mathbf{v}_2, \ldots, \mathbf{v}_m$ are linearly dependent. Then there exists $\lambda_1, \ldots, \lambda_n$ which are not all zero such that

$$\lambda_1 \mathbf{v}_1 + \cdots + \lambda_m \mathbf{v}_m = \mathbf{0}.$$

Say that $\lambda_i \neq 0$ and then we have

$$\mathbf{v}_i = -\frac{\lambda_1}{\lambda_i}\mathbf{v}_1 - \cdots - \frac{\lambda_{i-1}}{\lambda_i}\,\mathbf{v}_{i-1} - \frac{\lambda_{i+1}}{\lambda_i}\mathbf{v}_{i+1} - \cdots - \frac{\lambda_m}{\lambda_i}\mathbf{v}_m$$

as required.