Solution (#697) Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m$ be independent vectors in $\mathbb{R}^n$. Say that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots c_m\mathbf{v}_m = d_1\mathbf{v}_1 + d_2\mathbf{v}_2 + \cdots d_m\mathbf{v}_m$$

for real numbers c_i, d_i. Then

$$(c_1 - d_1)\mathbf{v}_1 + (c_2 - d_2)\mathbf{v}_2 + \cdots + (c_m - d_m)\mathbf{v}_m = \mathbf{0}.$$

By independence it follows that $c_i = d_i$ for each i as required.