Solution (#699) The answer is no. For example with $k = 2$, $l = 1 = m$ and choose

$$\mathbf{v}_1 = (1, 0)\,, \qquad \mathbf{v}_2 = (0, 1), \qquad \mathbf{w}_1 = (1, 0) \qquad \mathbf{x}_1 = (0, 1).$$