Solution (#703) Let A and $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{n-m}$ be as described in the exercise. By relabelling our variables if necessary, we may assume that the leading 1s appear in the first m columns of A. Consequently the $\mathbf{v}_i$ have the form

$$\begin{aligned}
\mathbf{v}_1 &= (-a_{1(m+1)}, -a_{2(m+1)}, \ldots, -a_{m(m+1)}, 1, 0, 0, \cdots, 0); \\
\mathbf{v}_2 &= (-a_{1(m+2)}, -a_{2(m+2)}, \ldots, -a_{m(m+2)}, 0, 1, 0, \cdots, 0); \\
& \quad \cdots \qquad \cdots \qquad \cdots \\
\mathbf{v}_{n-m} &= (-a_{1n}, -a_{2n}, \ldots, -a_{mn}, 0, , \cdots, 0, 1).
\end{aligned}$$

Focusing on the last $n-m$ entries, we can see that these vectors are clearly linearly independent.