Solution (#723) Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be k vectors in $\mathbb{R}^n$.

If for some i, $\mathbf{v}_i$ is in $\langle \mathbf{v}_1, \ldots, \mathbf{v}_{i-1} \rangle$ then we have

$$\mathbf{v}_i = \alpha_1 \mathbf{v}_1 + \cdots + \alpha_{i-1} \mathbf{v}_{i-1}$$

for some $\alpha_1, \ldots, \alpha_{i-1}$ and hence we have a non-trivial linear dependency

$$\alpha_1 \mathbf{v}_1 + \cdots + \alpha_{i-1} \mathbf{v}_{i-1} - \mathbf{v}_i = \mathbf{0}.$$

Conversely suppose that the $\mathbf{v}_i$ are linearly dependent so that we have an equation

$$\alpha_1 \mathbf{v}_1 + \cdots + \alpha_k \mathbf{v}_k = \mathbf{0}$$

where the α_i are not all zero. Let r be the largest integer such that $\alpha_r \neq 0$ and then we have

$$\alpha_1 \mathbf{v}_1 + \cdots + \alpha_r \mathbf{v}_r = \mathbf{0} \quad \Longrightarrow \quad \mathbf{v}_r = -\frac{\alpha_1}{\alpha_r} \mathbf{v}_1 - \cdots - \frac{\alpha_{r-1}}{\alpha_r} \mathbf{v}_{r-1}$$

so that $\mathbf{v}_r$ is in $\langle \mathbf{v}_1, \ldots, \mathbf{v}_{r-1} \rangle$.