Solution (#726) Let $\mathbf{a} = (a_1, \ldots, a_n) \neq \mathbf{0}$ so that the hyperplane has equation $a_1x_1 + a_2x_2 + \cdots + a_nx_n = c$. For it to contain the origin $(0, 0, \ldots 0)$ means that $c = 0$. Conversely if $c = 0$ and $\mathbf{v} = (v_1, v_2, \ldots, v_n)$, $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ are in the hyperplane with α, β are real constants then

$$\begin{aligned} & a_1(\alpha v_1 + \beta w_1) + a_2(\alpha v_2 + \beta w_2) + \cdots + a_n(\alpha v_n + \beta w_n) \\ = \; & \alpha(a_1v_1 + a_2v_2 + \cdots + a_nv_n) + \beta(a_1w_1 + a_2w_2 + \cdots + a_nw_n) \\ = \; & \alpha 0 + \beta 0 = 0, \end{aligned}$$

showing that $\alpha\mathbf{v} + \beta\mathbf{w}$ is in the hyperplane, and so the hyperplane is a subspace.

When $c = 0$ then the hyperplane is the solution space to the linear system

$$\left(\begin{array}{cccc|c} a_1 & a_2 & \cdots & a_n & 0 \end{array} \right)$$

which when reduced will have one leading 1 and $n - 1$ columns without a leading 1. Thus the solution space has dimension $n - 1$.