Solution (#742) Let $S = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_k\}$ and $T = \{\mathbf{t}_1, \mathbf{t}_2, \ldots \mathbf{t}_l\}$ be two finite subsets of $\mathbb{R}^n$. Then

$$S \cup T = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_k, \mathbf{t}_1, \mathbf{t}_2, \ldots \mathbf{t}_l\}$$

and

$$\begin{aligned} \langle S \cup T \rangle &= \left\{ \sum_{i=1}^{k} \alpha_i \mathbf{s}_i + \sum_{j=1}^{l} \beta_j \mathbf{t}_j \quad \text{for real scalars } \alpha_i, \beta_j \right\} \\ &= \left\{ \sum_{i=1}^{k} \alpha_i \mathbf{s}_i \quad \text{for real scalars } \alpha_i \right\} + \left\{ \sum_{j=1}^{l} \beta_j \mathbf{t}_j \quad \text{for real scalars } \beta_j \right\} \\ &= \langle S \rangle + \langle T \rangle. \end{aligned}$$

However $\langle S \cap T \rangle = \langle S \rangle \cap \langle T \rangle$ is not true. Let

$$S = \{(1,0), (0,1)\}, \qquad T = \{(1,0), (0,2)\},$$

with

$$\langle S \cap T \rangle = \langle (1,0) \rangle = x\text{-axis},$$

while

$$\langle S \rangle \cap \langle T \rangle = \mathbb{R}^2 \cap \mathbb{R}^2 = \mathbb{R}^2.$$