

**Solution** (#748) Let  $A$  be an  $m \times n$  matrix and  $B$  be an invertible  $n \times n$  matrix. Then

$$\begin{aligned}\text{rank}(AB) &= \text{rank}((AB)^T) && \text{as row rank equals column rank (Corollary 3.127)} \\ &= \text{rank}(B^T A^T) && [\text{by the product rule for transposes}] \\ &= \text{rank}(A^T) && \text{as } B^T \text{ is invertible (transpose rule) and by Proposition 3.88(d)} \\ &= \text{rank}(A) && \text{as row rank equals column rank (Corollary 3.127).}\end{aligned}$$