Solution (#749) Let $\mathbf{v}_1 = (1,1,2,1)$, $\mathbf{v}_2 = (2,4,4,1)$, $\mathbf{v}_3 = (0,2,0,-1)$. Note that

$$\mathbf{v}_2 - 2\mathbf{v}_1 = \mathbf{v}_3$$

and hence $\mathbf{v}_1$ and $\mathbf{v}_2$ span $\langle \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \rangle$. As $\mathbf{v}_1$ and $\mathbf{v}_2$ are clearly independent then $\mathbf{v}_1$ and $\mathbf{v}_2$ form a basis.

If we wished to be more algorithmic in our answer, we can put the three vectors as the rows of a matrix and row-reduce it. We then see

$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 4 & 4 & 1 \\ 0 & 2 & 0 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 2 & 3/2 \\ 0 & 1 & 0 & -1/2 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

and the non-zero rows form a basis.