Solution (#766) (i) The system is consistent if and only if $2y_1 = y_2 + y_3$.If this condition applies then the general solution is

$$(x_1, x_2, x_3, x_4) = (2y_1 - y_2 + \alpha + \beta, y_2 - y_1 - 2\alpha - \beta, \alpha, \beta).$$

(ii) The vector $\mathbf{y}$ is in $\mathrm{Col}(A)$ if and only if the system is consistent. As the columns of A span $\mathrm{Col}(A)$ then we can choose

$$(1, 1, 1)^T, \qquad (1, 2, 0)^T$$

as a basis.

(iii) If we set $\mathbf{y} = \mathbf{0}$ in (i) then we know the general solution has the form

$$(\alpha + \beta, -2\alpha - \beta, \alpha, \beta)$$

so a basis for the null space is

$$(1, -2, 1, 0), \qquad (1, -1, 0, 1).$$

This can alternatively be described as the solution space of

$$x_1 + x_2 + x_3 = 0, \qquad x_1 + 2x_2 + 3x_3 + x_4 = 0.$$