Solution (#782) From Example 3.134 we have

$$A_1 = \frac{1}{35}\begin{pmatrix} 59 & 32 \\ -18 & 11 \end{pmatrix}.$$

We saw that

$$A_1\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2.6 \\ -0.2 \end{pmatrix},$$

so that

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} + d\mathbf{v} = \begin{pmatrix} 2.6 \\ -0.2 \end{pmatrix},$$

where d is the distance of $(1,1)^T$ from the line $3x+4y=0$. From #518 this distance is

$$\frac{|3\times 1 + 4\times 1|}{\sqrt{3^2+4^2}} = \frac{7}{5}.$$

Hence

$$\frac{7}{5}\mathbf{v} = \begin{pmatrix} 1.6 \\ -1.2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 8 \\ -6 \end{pmatrix},$$

and hence

$$\mathbf{v} = \frac{1}{7}\begin{pmatrix} 8 \\ -6 \end{pmatrix} = \frac{2}{7}\begin{pmatrix} 4 \\ -3 \end{pmatrix}.$$

Remark: To better understand this shear we might introduce the vectors $\mathbf{w}_1 = (4,-3)/5$ and $\mathbf{w}_2 = (3,4)/5$. The vector $\mathbf{w}_1$ is a unit vector in the direction of the invariant line $3x+4y=0$ and $\mathbf{w}_2$ is a unit vector perpendicular to in the direction, so that

$$A_1(\mathbf{w}_1) = \mathbf{w}_1 \qquad \text{and} \qquad A\mathbf{w}_2 = \mathbf{w}_2 + \mathbf{v} = \mathbf{w}_2 + \frac{10}{7}\mathbf{w}_1.$$

Every vector in $\mathbb{R}_2$ can be written as $\alpha\mathbf{w}_1 + \beta\mathbf{w}_2$ for unique α, β. We then have

$$A_1(\alpha\mathbf{w}_1 + \beta\mathbf{w}_2) = \alpha\mathbf{w}_1 + \beta(\mathbf{w}_2 + \mathbf{v}) = \left(\alpha + \frac{10}{7}\beta\right)\mathbf{w}_1 + \beta\mathbf{w}_2.$$

The matrix for A_1 with respect to the basis $\mathbf{w}_1$ and $\mathbf{w}_2$ is

$$\begin{pmatrix} 1 & 10/7 \\ 0 & 1 \end{pmatrix}.$$

See §4.1 for further details.