Solution (#842) Note that we can rewrite the three given equations as

$$\begin{pmatrix} x & y & z \end{pmatrix} \begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix} = \mathbf{0}.$$

Note that the 3×3 matrix is a Vandermonde matrix. So if x, y, z are distinct then it is invertible and implies $x = y = z = 0$, a contradiction.

So we are left with the possibility that two (or more) of x, y, z are equal. Without loss of generality say $x = y$. Then we are left with the equations

$$2x + z = 2x^2 + z^2 = 2x^3 + z^3 = 0.$$

Substituting $z = -2x$ into the second two equations we get that $x = z = 0$ again as required.