

Solution (#846) (i) The row rank of the first matrix is 1, and any 1×1 submatrix is maximal and invertible,

(ii) The row rank of the second matrix is 2 and the maximal invertible 2×2 submatrices are

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}.$$

(iii) The row rank of the third matrix is 2 and the maximal invertible 2×2 submatrices are

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 3 & 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}, \quad \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}, \\ \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \\ \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix},$$