Solution (#851) (ii) Show that any $n \times n$ permutation matrix may be expressed as a product of elementary matrices S_{ij} by induction on n. If true of $(n-1) \times (n-1)$ permutation matrices use a transposition to move $\mathbf{e}_n$ from its current row to the nth row.

(iii) Let P be a permutation matrix and S_{ij} a transposition. Say that

$$P\mathbf{e}_I^T = \mathbf{e}_i^T, \qquad P\mathbf{e}_J^T = \mathbf{e}_j^T.$$

Show that $P^{-1}S_{ij}P = S_{IJ}$.